surfaces was higher than that of the glossy ones, and its sensitivity to the erosive action appeared to vary from one sample to another.

The c plane surface is also sensitive to oxidation, but less so than the a plane surface. This surface becomes more glossy with increased temperature and exposure time. However, the highest emittance measured corresponds to the highest temperature and the most glossy surface, indicating an influence of temperature that more than compensates for surface roughness effects. Similar observations are made in Ref. 11 (p. 471). A related observation made in Ref. 13 is that surfaces etched by heating in either vacuum or argon at temperatures above 2900°K had their luster restored by heating in oxygen. The work reported in the latter reference demonstrates the sensitivity of pyrolytic graphite emittance to surface conditions and environment of heating.

Concluding Remarks

Spectral hemispherical emittance and reflectance data have been obtained on a variety of ablation chars, carbon, and graphite at wavelengths of 0.4–3.2 μm and temperatures of 2200–3450°K. The spectral and integrated emittance data on carbon and polycrystalline graphite are in close agreement with comparable spectral and total emittance data in the literature. Total emittance data in the literature on ablation chars of the types evaluated are 10–20% lower than the integrated emittances. However, the literature data were obtained by methods based on assumed gray-body behavior of the materials, and when corrections based on the spectral definition obtained in the current study are made, the corrected values are only 2–10% lower than the integrated values.

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Uncertainties of Calculated Characteristics of a Transpiration-Cooled Arc

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Nomenclature

c_p = specific heat at constant pressure
E = electric field strength
h = enthalpy
H = mass average enthalpy
I = electric current
k = thermal conductivity
$\dot{m}$ = mass injection rate through porous wall
p = pressure
P_r = heat loss by radiation
r,z = coordinates
T = temperature
v = velocity
$\dot{w}$ = mass flow within constrictor in z direction
μ = viscosity
μ_0 = susceptibility of vacuum
ζ = bulk viscosity
ρ = gas density
σ = electrical conductivity

Subscripts

0 = on axis
p = in plenum chamber around outside wall
w = at constrictor wall

FOR the generation of a high-temperature, high-density plasma, transpiration-cooling of the constrictor tube in which the electric arc is operated has been suggested and corresponding experiments in this laboratory are already in progress.[1,2] From theoretical considerations, it seems possible to achieve higher axis temperatures with such an arc than with the conventional water-cooled cascaded arc.

In this work, the uncertainty of theoretical predictions, which suffer mainly from uncertainties of published values of the plasma transport properties, will be analyzed. The results of this study may be useful for other investigations as well which rely on the knowledge of the thermodynamic and transport properties of high-temperature plasmas.

Recently, Anderson described a method for the calculation of the characteristics of a transpiration-cooled arc[3] based on some earlier work of Anderson and Eckert.[4] In connection with analyzing the performance capability of a transpiration-cooled constricted arc heater, Anderson and Eckert used this method successfully.[5] Assuming a thermally and hydrodynamically fully developed laminar flow, the continuity,

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momentum, and energy equations are

$$\frac{1}{r}\frac{\partial r\rho v_r}{\partial r} + \rho\frac{\partial v_z}{\partial r} = 0 \qquad (1)$$

$$\rho v_z \frac{\partial v_z}{\partial z} + \rho v_r \frac{\partial v_z}{\partial r} + \frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v_z}{\partial r}\right) \qquad (2)$$

$$\frac{\partial p}{\partial r} = -\rho v_r \frac{\partial v_r}{\partial r} + 2\frac{\partial}{\partial r}\left(\mu\frac{\partial v_r}{\partial r}\right) + \mu\frac{\partial}{\partial r}\left(\frac{\partial v_z}{\partial z}\right) + \frac{2\mu}{r} \times \left(\frac{\partial v_r}{\partial r} - \frac{v_r}{r}\right) - \frac{2}{3}\frac{\partial}{\partial r}(\mu \operatorname{div}\mathbf{v}) + \frac{\partial}{\partial r}(\zeta \operatorname{div}\mathbf{v}) - \frac{\mu_0 D}{2\pi}\sigma\frac{I(r)}{r} \qquad (3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) - \rho v_r\frac{\partial h}{\partial r} + \sigma E^2 - P_r = 0 \qquad (4)$$

with the boundary conditions

$$v_r(0) = 0,\ v_r(r_w) = -\dot{m}/\rho$$

$$(dT/dr)(0) = 0,\ v_z(r_w) = 0,\ (dv_z/dr)(0) = 0$$

$$-k\left.\frac{dT}{dr}\right]_{r=r_w} + \frac{1}{2\pi r_w}\int_0^{r_w} P_r 2\pi r dr = \dot{m}(h_w - h_p)$$

The derivation and the numerical solution of these equations are discussed in Ref. 3. In order to obtain reliable quantitative results, the transport and thermodynamic properties of the gases, which are used for cooling, have to be well known. Unfortunately, the values collected in the literature, especially those of the transport properties and the volumetric radiation, differ very much. In Fig. 1 the temperature dependence of the thermal conductivity of argon is shown as reported by four different authors.[6–8,12] The volumetric radiation emitted by an argon plasma as a function of the temperature is plotted in Fig. 2 with reference to two different authors.[7,9] The deviations between the values of the individual authors at a given temperature are very large. For example, the difference between the thermal conductivities at 13,500°K reported in Refs. 6 and 7 is nearly an order of magnitude. An examination of the transport and radiation properties for nitrogen reported in the literature revealed a disagreement in the same order of magnitude. Some of the disagreement in the thermal conductivity and radiation properties is apparently caused by the fact that some are derived from experimental data on optically thick columns and others from kinetic formulations. It is, however, not clear today which of the properties should be used in the analysis.

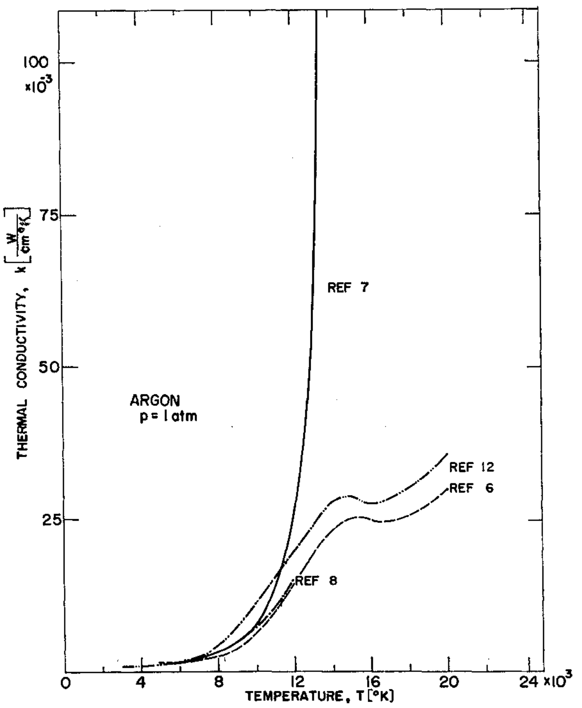

Fig. 1 Thermal conductivity of an atmospheric argon plasma as a function of temperature.

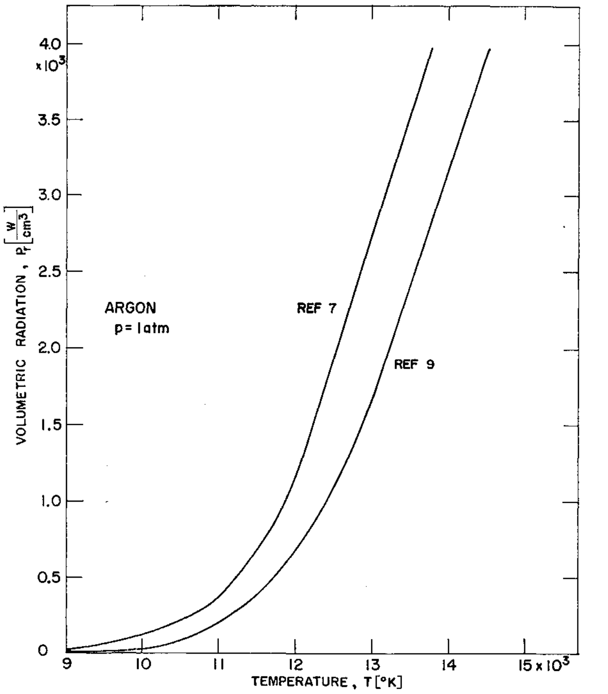

Fig. 2 Volumetric radiation of an atmospheric argon plasma as a function of temperature.

The effect of these uncertainties of the plasma transport properties on the results of calculated arc characteristics and temperature profiles is demonstrated in the following graphs and tables. In Fig. 3, three different radial temperature distributions for a transpiration-cooled argon arc are shown. The comparison in each one of these and those shown in Fig. 4 is made for a fixed peak temperature and a fixed wall temperature. The thermodynamic and transport properties used in the numerical solutions have been obtained as follows. Case A: ρ, c_p, h from Ref. 10; k, σ, μ from Ref. 6; $P_r = 0$. Case B: ρ, c_p, h from Ref. 10; k, σ from Ref. 8; μ from Ref. 6;

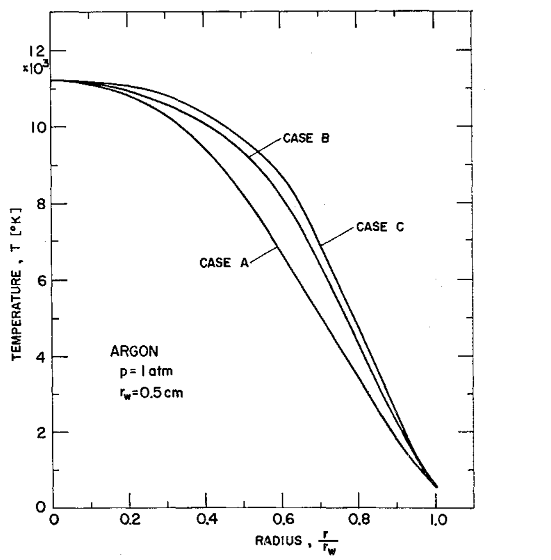

Fig. 3 Influence of uncertainties in the thermal conductivities on the temperature profiles of a transpiration-cooled arc.

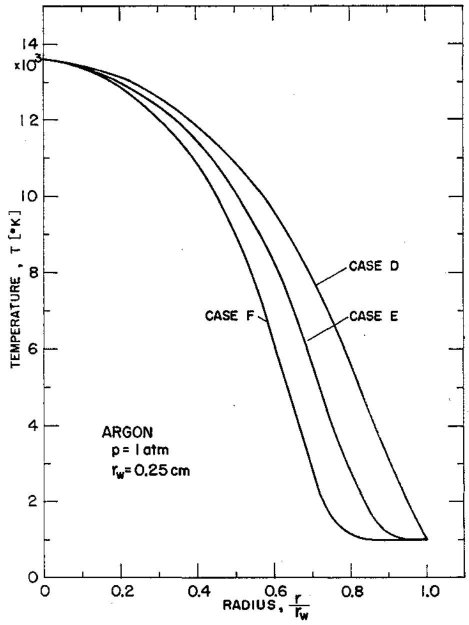

Fig. 4 Influence of uncertainties of the volumetric radiation on the temperature profiles of a transpiration-cooled arc.

P_r from Ref. 9. Case C: ρ, c_p, h from Ref. 10; k, σ, P_r from Ref. 7; μ from Ref. 6.

For the low-temperature range ($T < 3{,}000°K$), the gas properties from Ref. 11 have been used. Since the peak temperature is relatively low, the influence of the volumetric radiation is almost negligible in this range. In addition, the values of the electrical conductivity used for the calculations are in much better agreement from author to author than those of the thermal conductivity. Therefore, the spread of the different calculated temperature profiles is caused mainly by uncertainties of the thermal conductivities. The effect of this uncertainty on the arc characteristics is demonstrated in Table 1, which contains calculated values of the electric field strength, the electric current, the power input, the mass injection rate through the porous wall, and the mass average enthalpy for cases A, B, and C.

The influence of the not-well-known volumetric radiation on the arc characteristics and the temperature profiles is evident from Fig. 4 and Table 2. The calculations are based on the following gas properties. Case D: ρ, c_p, h from Ref. 10; k, σ, μ from Ref. 6; $P_r = 0$. Case E: ρ, c_p, h from Ref. 10; k, σ, μ from Ref. 6; P_r from Ref. 9. Case F: ρ, c_p, h from Ref. 10; k, σ, μ from Ref. 6; P_r from Ref. 7.

In this case the peak temperature reached the value of 13,600°K and the temperature at the wall was 1,000°K. Neglecting the volumetric radiation entirely or using the values given in Ref. 7 leads to deviations in the temperature profiles of up to 5000°K. The disagreement of the arc characteristics is shown in Table 2.

From these examples which have been taken from a large number of calculations, it is obvious that the uncertainties in the plasma transport properties cause large uncertainties of the temperature distributions and arc characteristics of a transpiration-cooled arc. Because of this problem, a quantitative comparison of analytical and experimental results is presently nearly impossible. Comparisons can be undertaken only to demonstrate agreement of basic trends.

Table 1 Influence of the thermal conductivity on arc characteristics

Case	E, v/cm	I, amp	EI, kw/cm	$\dot{m}$, g/cm² sec	H, kjoules/kg
A	4.25	36.5	0.155	0.01841	2687
B	5.37	53.0	0.285	0.02669	3400
C	5.39	59.8	0.322	0.02854	3595

Table 2 Influence of the volumetric radiation on arc characteristics

Case	E, v/cm	I, amp	EI, kw/cm	$\dot{m}$, g/cm² sec	H, kjoules/kg
D	10.64	42.6	0.453	0.0531	5431
E	13.12	41.2	0.54	0.0821	3835
F	15.13	37.3	0.57	0.1009	3231

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Light Antiaircraft Projectile Ballistics

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Nomenclature

$\bar{A}$ = reference area for drag coefficient
C_D = drag coefficient
$\bar{D}$ = $C_D \frac{1}{2}\bar{\rho}\bar{v}^2\bar{A}$ = aerodynamic drag force
$\bar{g}$ = acceleration of gravity
$\bar{m}$ = projectile mass
$\bar{S}_D$ = $2\bar{m}/C_D\bar{\rho}\bar{A}$ = aerodynamic penetration
$\bar{t}$ = time
$\bar{v}$ = projectile velocity
$\bar{y}$ = horizontal (downrange) coordinate
$\bar{z}$ = vertical (altitude) coordinate

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